Technology Tandem

GSDMC High School Math Field Day

March 14, 2009

Sponsored by:

Cuyamaca College
San Diego City College

1. Find the prime factorization of $456,288$

Solution: $2^5 \cdot 3 \cdot 7^2 \cdot 97$

2. Find the smallest pair of twin primes greater than 400.

Solution: $419, 421$

3. Find the smallest pair of twin primes greater than 500.

Solution: $521, 523$

4. Find the prime factorization of $250,538,275$.

Solution: $5^2 \cdot 13^2 \cdot 19 \cdot 3121$

5. What is the Least Common Multiple for $5,525$ and $10,718,825$

Solution: 182220025

6. Find the Greatest Common Factor for $3,275$ and $1,179$

Solution: 131

7. Find the Greatest Common Factor for $3,725$, $5,364$, and $1,341$.

Solution: 149

8. Find the Greatest Common Factor for $47,250$ and $456,225$

Solution: 525

9. Solve for x (to the nearest hundredth)

$$4\sin(50x) = 10\cos(x) + \frac{\tan(40x!)}{x-1} + 19900$$

Solution: 1.00

10. Solve for x (to the nearest hundredth)

$$\frac{4x! - \frac{3}{2x^2} + 50x^4}{100x + 5x^2} + 300x = 1000$$

Solution: 3.22

11. Solve the system (to the nearest hundredth)

$$\sin(x+y) - e^x \cdot y = 0$$
$$x^2 - y = 2$$

Solution: $x = -6.02, y = 34.21$

12. Solve $(x^4 - 2x + 4)^{x^3-3x+4} = 1$

Solution: -2.20

13. Solve $\sqrt{x}^{\sqrt{x}} - 5 = 120$

Solution: 13.65

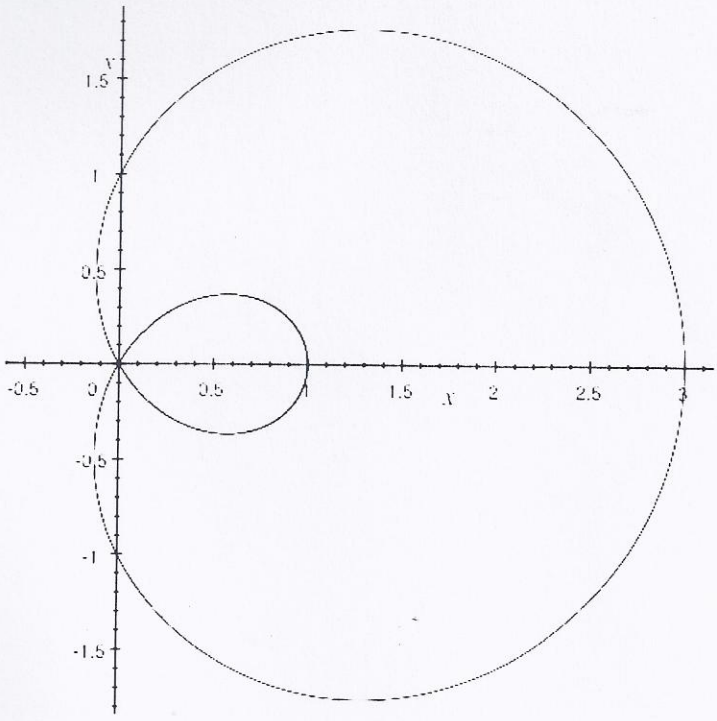
14. Find the exact value of $\sum_{k=1}^{\infty} \frac{3k}{2^k}$

Solution: 6

15. Find the exact value of $\sum_{k=1}^{\infty} \frac{k^3}{k!}$

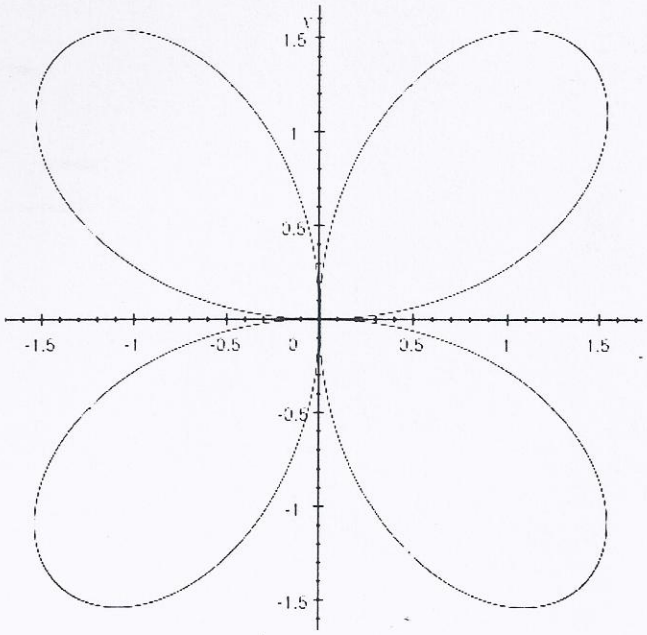
Solution: $5e \approx 13.59$

16. Make a plot of a limaçon shown below. Show the graph to an instructor and be sure to set the graphing window and axes scales so that the instructor can readily see the size of the graph.



Solution: $r(\theta) = 2\cos\theta - 1$

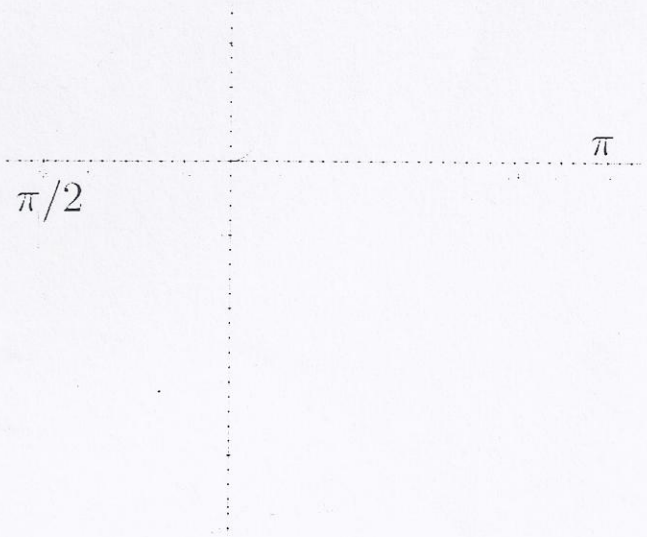
17. Make a plot of a flower with 4 petals shown below. Show the graph to an instructor and be sure to set the graphing window and axes scales so that the instructor can readily see the size of the graph.



Solution: $r(\theta) = 2\sin\theta$

18. Make a plot of the spiral shown below. Show the graph to an instructor and be sure to set the graphing window and axes scales so that the instructor can readily see the size of the graph.

Solution: $r(\theta) = \frac{1}{2}\theta$



The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ can be determined by evaluating the following determinant:

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Where the $\pm$ symbol indicates that the appropriate sign should be chosen to yield a positive area. Find the areas of the following triangles, determined by the given points. Round the answers to the nearest tenth if necessary.

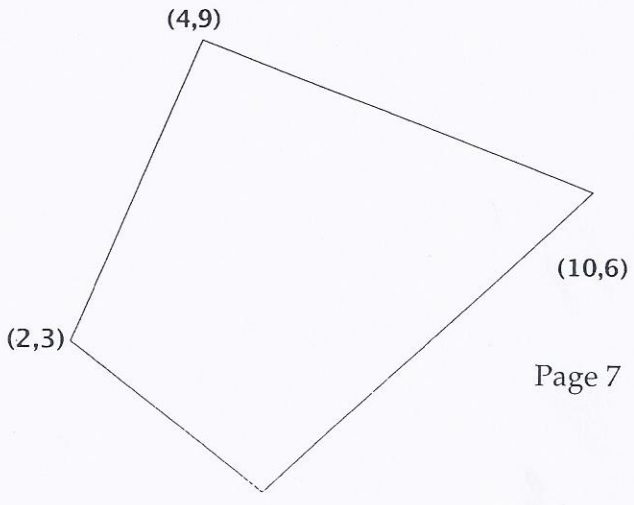
19. Triangle's vertices: $(-4, -3), (5, 3), (12, -11)$

Solution: 84

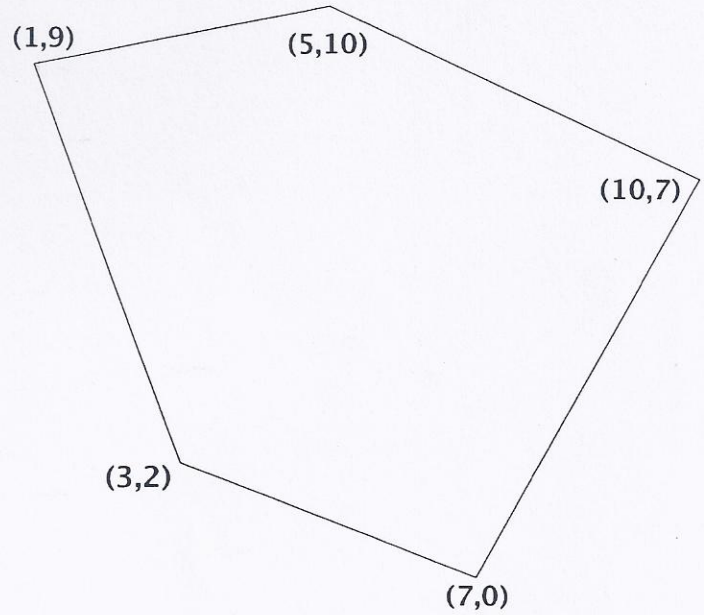
20. Triangle's vertices: $(\sqrt{7}, e^4), (-\sqrt[3]{4}, 1/4), (11, e^2)$

Solution: 326.9

21. Find the area of the region defined by the vertices given below

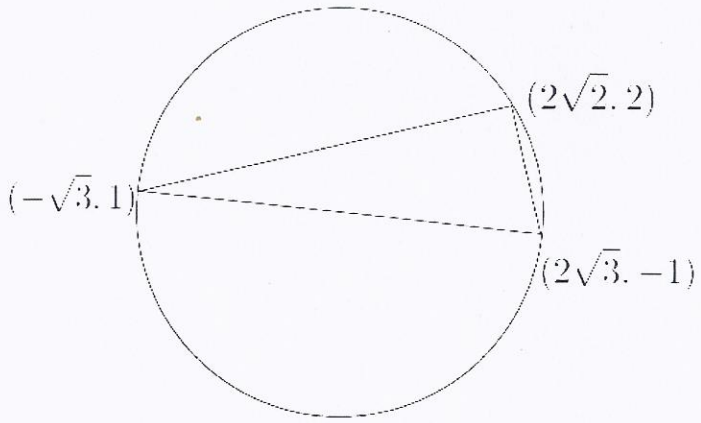


22. Find the area of the region defined by the vertices given below



Solution: 55

23. Find the area of the shaded region below



Solution: 17.2

24. Find all values of x for which the triangle with the following vertices has an area of 15: $(-3, -4), (x, 2), (2, 6)$

Solution: 3

Statistics

Without using the variance and standard deviation function of your graphing calculator, we'll find tha variance and standard deviation for the following data observations.

2.5	4.7	12.1	5.2	8.9	12.5	11.5	12.9	5.4	12.7	15.9	10.2
6.5	16.1	1.5	4.8	19.0	0.4	12.4	11.9	8.4	5.3	7.9	1.1
3.2	1.5	7.3	8.2	6.2	9.3	14.2	11.9	19.7	50.0	3.2	1.9

25. Create a list, L1, of these data, and se the list to find the mean, $\bar{x}$, for these data (round to the nearest thousandth if necessary).

Solution: 9.622

26. Create a second list, L2, of the deviations from the mean; i.e. calculate ?, for each observation, ?, in L1 and store these deviations in L2. Sum L2 (round to the nearest thousandth if necessary).

Solution: 0

27. Create a third list, L3, of the squares of the deviations in L2; i.e. calculate? for each observation, ?. To find the variance for the original data observations, find the arithmetic mean of the entries in L3. Note: our variance may differ from applying the graphing calculators variance function to the original data observations. (Round to the nearest thousandth if necessary).

Solution: 72.188

28. The standard deviation,*sd* for the original data observations is the square root of the variance found in the previous problem. Find the standard deviation, sd (round to the nearest thousandth if necessary).

Solution: 8.496

29. The z-score (or standard score) for any given observation, x?, is $\frac{x-\bar{x}}{sd}$?. Create a fourth list, L4, of the z-scores for the original data observations. What is the maximum z-score in the list (rounded to the nearest thousandth, if necessary).

Solution: 4.752

Algebra

Unless you are directed to do otherwise, round irrational numbers to the nearest hundredth. Do not round rational numbers.

30. Use your calculator to find the equation of the quadratic function that passes through the points $(-4, 5), (-2, -10), (12, 100)$

Solution: $y = .96x^2 - 1.74 - 17.32$

31. Use your calculator to find the equation of the exponential function passing through the points $(2, 0.54), (4, 0.0486), (5, 0.01458)$.

Solution: $y = 6 \cdot (0.3)^x$

32. Solve the following system of equations (write rational solutions in fractional notation, and round irrational solutions to the nearest tenth).

$$2x + 5y = 4$$
$$7x - 3y = 12$$

Solution: $x = 72/41, y = 4/41$

33. Solve the following system of equations (write rational solutions in fractional notation, and round irrational solutions to the nearest tenth).

$$7x - \tfrac{1}{2}y - \tfrac{1}{3}z = 2$$

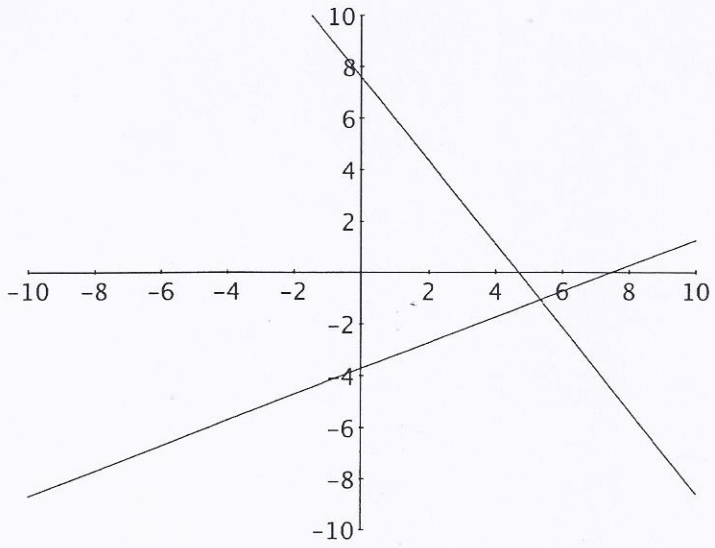
$$\tfrac{3}{2}x - 7y - \tfrac{5}{3}z = 12$$

$$\tfrac{4}{3}x + \tfrac{5}{12}y - 3z = 1$$

Solution: $x = 1600/10613, y = -16628/10613, z = -5136, 10613$

34. Use your calculator to graph the solution set to the following system of inequalities. Show the graph of the solution set to the instructors at the front of the room.

$$2y - x \geq -7$$
$$2y + 3x \leq 15$$
$$y \leq 0$$
$$x \leq 0$$



35. During the first 13 seconds of a jump, a skydiver falls approximately $11.12t^2$ feet in t seconds. A small heavy object (with less wind resistance) falls about $15.4t^2$ feet in t seconds. Suppose that a skydiver jumps from $30,000$ ft, and 1 sec later a camera falls out of the airplane. How long will it take the camera to catch up to the skydiver?

Solution: 5.7 seconds

36. Solve the inequality $2.3x^2 - 20.5x > 5$ and write the answer in interval notation.

Solution: $(-\infty, -0.24) \cup (9.15, \infty)$

37. Solve the inequality below and write the answer in interval notation.

$$\frac{2x^2 + 4.5x - 1.5}{2x + 3.1} > 7$$

Solution: $(-1.78, -1.55) \cup (6.53, \infty)$

38. Solve the inequality $5.23x^2 - 20.5x < 105.1x^2 + 40.3x - 11$ and write the answer in interval notation.

Solution: $(-1.78, -1.55) \cup (0.15, \infty)$

39. Sales of DVD players have grown from \$171 million in 1997 to \$1099 million in 1999 and \$2697 million in 2001. Find the exponential function that models the total sales of DVD players t years after 1997.

Solution: $y = 200.76 \cdot 1.99^t$

40. Find the points of intersection of the graphs below

$$x^2 + 3y^2 = 4$$
$$2x - 3y^2 = 2$$

Solution: $(1.65, -0.66), (1.65, 0.66)$

41. Find the points of intersection of the graphs below

$$0.3x^3 - e^y = 4$$
$$5x^3 - 2y^2 = 12$$

Solution: $(2.37, -0.23)$

42. Find all real solutions to the equation $0.1x^4 + 3.2x^3 - 4x^2 + 3x = 0$.

Solution: $x = 0, -32.15$

43. Factor the following polynomial completely using only integers

$$105x^4 - 464x^3 + 453x^2 - 10x - 24$$

Solution: $(x-3)(3x-4)(5x+1)(7x-2)$

Use the matrices defined below for the following four problems. Do not use decimal notation in your matrix entries. Use fraction notation for rational numbers (excluding integers).

$$M = \begin{bmatrix} 2 & -5 & 7 \\ 4 & -2 & 11 \\ 12 & 3 & 11 \end{bmatrix} \qquad N = \begin{bmatrix} -5 & 15 & 1 \\ 3 & -1 & 21 \\ 1 & 9 & -5 \end{bmatrix}$$

44. Find MN

Solution: $MN = \begin{bmatrix} -18 & 98 & -136 \\ -15 & 161 & -89 \\ -40 & 276 & 32 \end{bmatrix}$

45. Find NM

Solution: $MN = \begin{bmatrix} 74 & 1 & 152 \\ 254 & 50 & 241 \\ -22 & -38 & 51 \end{bmatrix}$

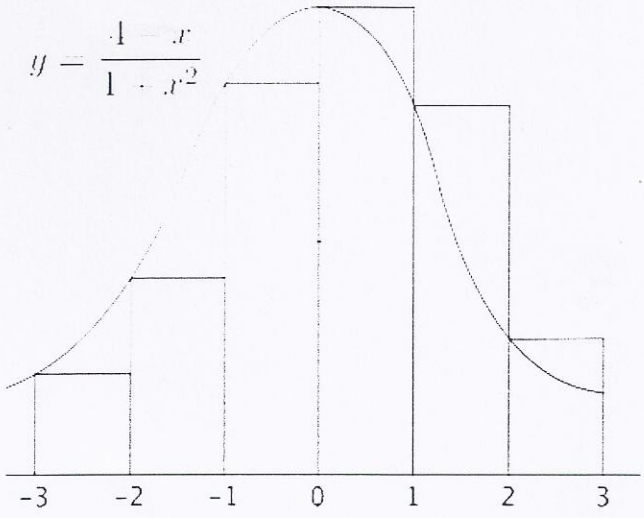
46. If $f(x) = 3x^4 - 5x^3 + 7x^2 + 7x - 3$, evaluate $f(M)$

Solution: $f(M) = \begin{bmatrix} 37559 & 3273 & 35835 \\ 89340 & -1723 & 99837 \\ 127116 & -14571 & 160088 \end{bmatrix}$

47. Find the multiplicative inverse of matrix M (remember to use fractions - not decimals)

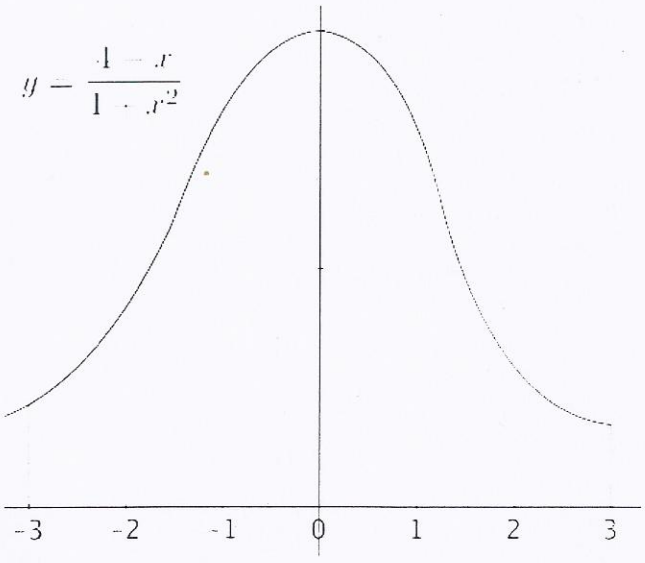
Solution: $M^{-1} = \begin{bmatrix} 55/298 & -38/149 & 41/298 \\ -44/149 & 31/149 & -3/149 \\ -18/149 & 33/149 & -8/149 \end{bmatrix}$

48. Find the area of the shaded region below (leave in fraction notation)



Solution: 103/10

49. Find the area underneath the shaded region below (round to the nearest thousandth).



Solution: 9.992